LEVEL ⑩

AD-
FINAL REPORT
IRO REPORT NO. 263

INTEGRATED FORECASTING TECHNIQUES FOR SECONDARY ITEM CLASSES
PART II - INACTIVE ITEMS

U.S. ARMY

INVENTORY

RESEARCH

OFFICE

September 1980

ROOM 800
U.S. CUSTOM HOUSE
2nd and Chestnut Streets
Philadelphia Pa. 19106

81 3 30 001

(14) IRO-263-PT-2

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER IRO Report No. 263	2. GOVT ACCESSION NO. AD-A097 025	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) INTEGRATED FORECASTING TECHNIQUES FOR SECONDARY ITEM CLASSES - PART II - INACTIVE ITEMS.		5. TYPE OF REPORT & PERIOD COVERED FINAL REPORT.
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) EDWIN P. GOTWALS, III		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Inventory Research Office, ALMC Room 800, US Custom House 2nd & Chestnut Streets, Philadelphia, PA 19106		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Materiel Development & Readiness Command 5001 Eisenhower Avenue Alexandria, VA 22333		12. REPORT DATE September 1980
		13. NUMBER OF PAGES 58
14. MONITORING AGENCY NAME & ADDRESS(If different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for Public Release; Distribution Unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Information and data contained in this document are based on input available at the time of preparation. Because the results may be subject to change, this document should not be construed to represent the official position of the US Army Materiel Development & Readiness Command unless so stated.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Demand Forecasting
Low Demand
SPORATIC Demand
Army Parts
Error Measures

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report describes the empirical development and testing of demand forecast algorithms specifically designed for inactive items. The data consisted of a sample from approximately 20,000 aviation repair parts. The intent was to improve demand forecasts at the wholesale level. Included is a description of the efforts to develop characteristics of these items, to develop appropriate forecast algorithms based on these characteristics and to evaluate the algorithms in an inventory management setting.

DD FORM 1473 EDITION OF 1 NOV 65 IS OBSOLETE
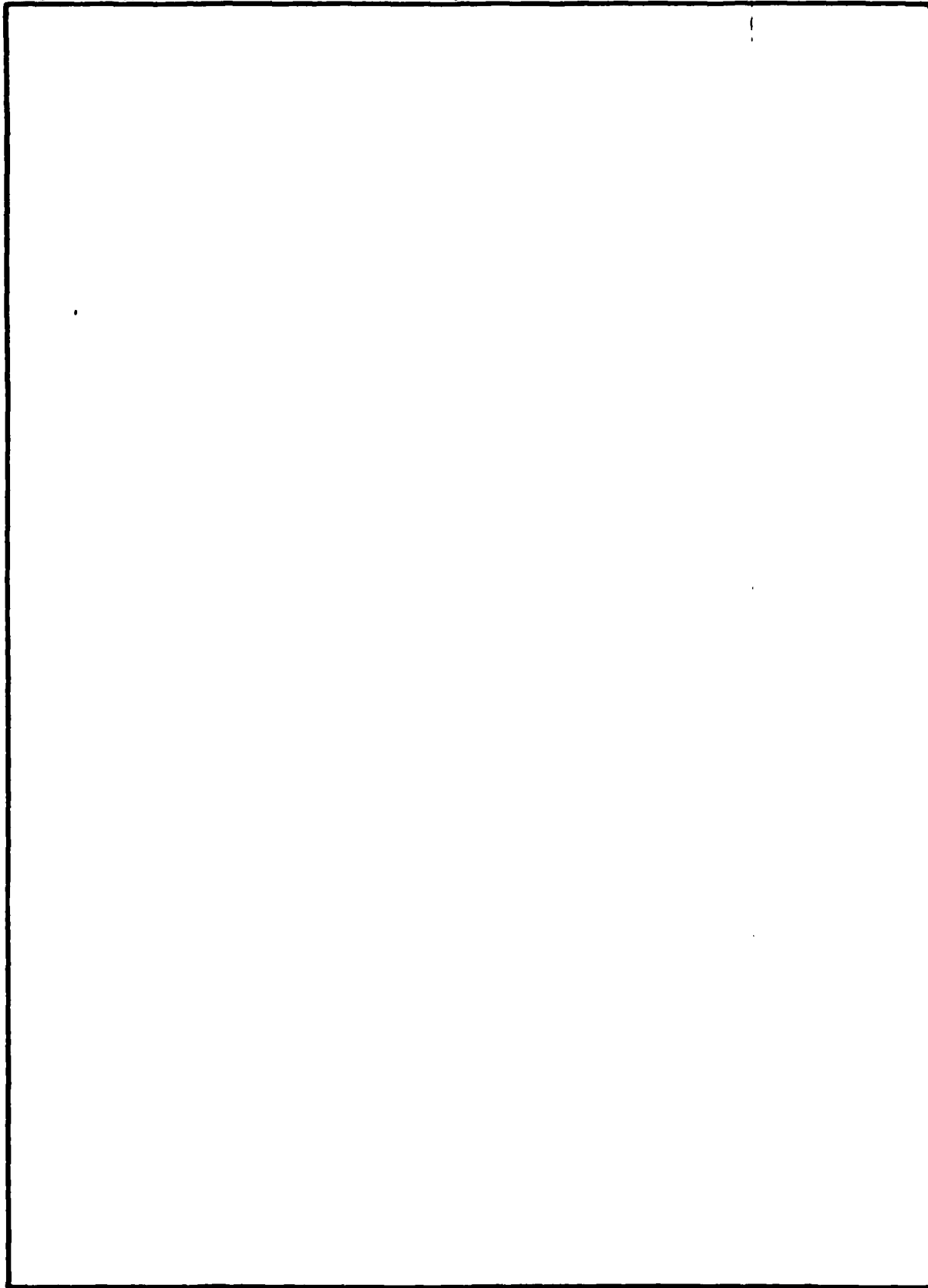
SUMMARY

1. Overview

The Army Inventory Research Office has been involved in several forecasting studies with the intent of improving the forecast of demand for replacement parts in the Army wholesale inventory system. This report describes the work done with those items which are stocked for various reasons but exhibit very little demand. It was felt that by studying the characteristics of these items, a forecast algorithm could be developed which would work better than traditional methods when applied in any inventory setting.

The work consisted of data experiments performed on a data base of 11 years of quarterly demands for approximately 20,000 aviation replacement parts. Of these, nearly 80% were classified as inactive.

2. Results

a. Descriptive variables such as maintenance factors, inventory management processing codes, unit price, and procurement lead time could not be used to characterize low demand items.

b. Many of the inactive items had short interarrival times between requisitions leading to the inference that the requisitions arrive in clusters.

c. The demands per requisition increased as the requisition frequency increased.

d. The data analysis did not clearly show a dependent demand pattern in the state series. The state series $\{y_t\}$ is defined in the following manner:

$$y_t = \begin{cases} 1, & x_t > 0 \\ 0, & x_t = 0 \end{cases}$$

where $\{x_t\}$ is the demand series

e. Plots of certain conditional probabilities indicated that zeros tended to follow zeros and ones tended to follow ones but statistical tests did not confirm this.

f. Three expected-value forecast algorithms were developed by making various model assumptions based on the data analysis results. The models considered were: an independent demand model, a first order dependence model and an inter-arrival time model.

1

g. The dependent model showed a 5.5% increase in performance in terms of requisitions satisfied at the end of a lead time.

2

TABLE OF CONTENTS

4

CHAPTER I

INTRODUCTION

1.1 Overview

This report is a compilation of experiences gained while trying to improve forecasts for low demand items. It is based on experimentation done on a data base of approximately 20,000 aviation repair parts. This experimentation was conducted in conjunction with IRO Project 263 "Integrated Forecasting Techniques for Secondary Item Classes" in an effort to improve demand forecasts for all items at the wholesale level. The observations cited herein describe the efforts to determine characteristics of low demand items, to develop appropriate forecast algorithms based on these characteristics and to evaluate these algorithms in an inventory environment. It is hoped that these empirical findings will prompt additional research in these areas.

1.2 Background

Over the past few years, the Inventory Research Office (IRO) has been involved in several forecast projects designed to improve the forecast of demand for replacement parts in an automated wholesale supply system in operation at the Army Commodity Commands. In the earlier work [7] forecast algorithms were developed and tested for use on all items independent of the demand activity of the individual item. In later work [10] items were stratified into classes according to requisition frequency and dollar demand with the hope of finding optimal forecast techniques for each class. The work presented here describes the details of the analysis done on the class of low demand or inactive items.

1.3 Data

The demand history file used in this study consists of 11 years of requisitions and demands by quarter accumulated from the Troop Support and Aviation Materiel Readiness Command's (TSARCOM) Demand, Return and Disposal file. The items were limited to aviation parts for which flying hours could be obtained. The file contains a sample of 20,865 items from those which were in the system between 1966 and 1977.

The demands are limited to parts subject to demand forecasting in the Army inventory management process. They include worldwide recurring demands for Army managed class IX secondary items (repair parts and spares). (Recurring demand is that portion of total demand which is representative of a continuing demand process.)

Each item is classified as low dollar value (LDV) or high dollar value (HDV) according to whether the demand rate averaged over the 11 years was less than or greater than or equal to $50,000. Items with over a million dollars of demand per year were dropped.

The items were further divided into dynamic (DYN) and non-dynamic (NON) based on the Federal Supply Classification (FSC). The dynamic components were considered to be those that experience high rotation rates; i.e. rotor blades, transmissions, and turbine engines. For more detail see Cohen [2].

The data breaks out into the following four groups by number of items:

HDVDYN	86
HDVNON	262
LDVDYN	1169
LDVNON	19348
	20865

1.4 Organization

The objectives of the study are as follows:

a. Identify an "inactive item" using only classification data (prior to demand experience).

b. Determine statistical characteristics of the demand for these items.

c. Develop forecast techniques which will depend on the inherent nature of this class of items.

d. Evaluate the forecast techniques using a statistical error measure developed for this class of items, and determine a best forecast algorithm based on statistical and simulation results.

Chapters II through V cover the four objectives respectively. Chapter VI summarizes the conclusions from each chapter and suggests areas for future research.

CHAPTER II

IDENTIFICATION

2.1 Methodology

In the previous IRO studies the demand activity of an item was measured by the average number of requisitions per year. Assuming that this is a reasonable measure, each item's average requisition per year was compared with other item variables in an effort to:

a. Identify properties of the low demand items that might be used to improve the demand forecast.

b. Identify a low demand item when little or no demand history is available.

2.2 Variable Description

The variables being considered are separated into two classes, demand and descriptive, and are defined as follows:

Demand Variables

AVG REQ PER YEAR (REQ Class)	=	The average number of requisitions per year observed during the active* portion of the series.
AVG DEMANDS PER YEAR (DMD Class)	=	The average number of demands per year observed during the active* portion of the series.
INTERARRIVAL TIMES	=	The average time between requisitions starting with the first and ending with the last.

Descriptive Variables

IMPC Code	=	Inventory Management Processing Code, a code designed to indicate the status and supply policy for the item.
UNIT PRICE	=	The price charged to the customer for the item.
MAINTENANCE FACTOR	=	An engineer estimate of the expected number of failures for a given part per 100 end items in one year.
PROCUREMENT LEAD TIME	=	An estimate of the time it takes to receive 1/3 of the order once the buy decision is made.

---

*Double 12 month moving average starting after the first non-zero demand.

2.3 Observations

The following inferences may be drawn from the tables at the end of this chapter where the lower requisition classes (up to 6 req. per year) are cross tabulated against each of the other variable classes.

Requisition vs Demand: As the requisition frequency increased so did the requisition size, i.e., in class [0,1), 60% of the items had one demand per requisition, whereas in req class [5,6) only 5% averaged this low. Consequently the demand per requisition ratio is requisition class dependent and unit requisition size cannot be assumed.

Requisition vs Interarrival Time: Low average interarrival times (1,2,3) dominated all requisition classes including the lowest demand classes. This indicates a possible clustering effect, i.e. several back to back requisitions during the active portion of the series (e.g. (0,0,0,1,1,1,0,0,0) = 1 quarter interarrival time).

Requisition vs IMPC Codes: 82% of all items had an average requisition frequency of less than 6 per year and of these -

           25% were stocked
           21% were non-stocked
           20% were semi-active
           13% were insurance items
            9% were terminal

Also 15.5% of all items were stocked and had average requisitions less than 3 per year.

Requisition vs Unit Price: Unit price was distributed approximately the same throughout each requisition class.

Requisition vs Maintenance Factor: Low Maintenance Factors dominated all the requisition classes.

2.4 Conclusion

For low requisition class items, the demands may occur in clusters and the number of demands per requisition is related to requisition frequency. None of the descriptive variables can adequately predict the requisition activity of the item.

Requisitions vs Demands

Avg Yearly Demands	Requisition Classes*						
	Trivial	[0,1)	[1,2)	[2,3)	[3,4)	[4,5)	[5,6)
[0,1)	7956	1608					
[1,2)	981	598	751				
[2,3)	296	176	621	347			
[3,4)	130	79	276	345	188		
[4,5)	62	37	156	204	184	73	
[5,6)	49	35	100	126	155	97	33
[6,7)	50	20	74	90	107	82	54
[7,8)	27	6	42	58	71	69	57
[8,9)	25	8	38	40	44	51	56
[9,10)	15	5	29	51	40	34	44
[10,20)	59	34	107	160	144	178	206
[20,30]	23	15	38	38	39	50	57
(30,40]	10	2	12	25	26	23	32
(40,50]	8	2	14	8	7	11	23
(50,60]	3	1	3	2	6	8	12
(60,70]	0	1	5	3	4	9	5
(70,80]	4			3	4	3	3
(80,90]	1		1	3	2	1	4
>90	6	7	12	12	17	11	19
% of Items with more than 1 demand per Requisition		40%	67%	76%	82%	90%	95%

* Average number requisitions per year
  Trivial Class; Each calendar year had <4 requisitions

Requisitions vs Inter-Arrival Times

(qtas) Inter-Arrival Time	Requisition Classes						
	Trivial	[0,1)	[1,2)	[2,3)	[3,4)	[4,5)	[5,6)
0							
1	658	851	621	655	730	622	583
2	944	740	946	722	299	74	24
3	945	402	433	107	8	6	
4	809	254	144	21	1		
5	473	132	51	7	2		
6	383	93	17	1			
7	306	55	8	2			
8	242	19	3				
9	203	13	4				
10	155	15	1				
11	108	6					
12	83	2					
13	77	3					
14	74	5					
15	63	1					
> 16	247	5	1				

10

Requisitions vs IMPC Codes

IMPC Codes	Requisition Classes						
	Trivial	[0,1)	[1,2)	[2,3)	[3,4)	[4,5)	[5,6)
AA	1307	366	372	212	110	79	54
1B		1				2	1
1C	1537	381	810	183	566	395	351
3A	2556	512	470	243	119	51	39
6B	359	111	53	35	21	17	13
6C	2	1	1	1		1	
6D	19	5	1	1			
6E	49	28	8	1	3	1	1
6F	250	65	16	11	2	2	2
6J	102	69	31	20	9	12	7
6P	1					1	
9C	944	401	159	117	85	63	57
9D	136	111	50	20	11	3	5
9X	2443	584	304	171	111	75	77

Requisitions vs Unit Price

$ Unit Price	Requisition Classes						
	Trivial	[0,1)	[1,2)	[2,3)	[3,4)	[4,5)	[5,6)
[0,.01]	6	1	2		1		
(.01,.10]	67	13	10	10	3	1	5
(.1,1.0]	3382	839	709	438	285	181	198
(1.0,10]	4002	1161	996	663	447	317	255
(10,100]	1835	498	452	326	241	142	111
(100,1000]	362	114	104	74	56	40	37
(1000,10000]	51	8	6	4	7	1	2
(10,000,100,000]		1					

Requisitions vs Unit Price

$Unit Price	Requisition Classes						
	[6,7)	[7,8)	≥ 8				
[0,.01)		1					
(.01, .10]		1	13				
(.10, 1.]	128	115	879				
(1.0, 10]	193	159	1267				
(10, 100]	110	87	873				
(100, 1000]	30	26	252				
(1000, 10000]	2	2	43				
(10000, 100000]	1	1	5				

Requisitions vs Maintenance Factors

Maintenance Factor	Requisition Classes						
	Trivial	[0,1)	[1,2)	[2,3)	[3,4)	[4,5)	[5,6)
0	4929	1018	790	524	325	216	178
1	3386	1217	989	638	448	292	249
2	282	101	95	57	46	28	35
3	163	71	64	43	32	37	20
4	25	4	10	5	4	3	4
5	25	22	15	16	21	10	8
6	14	7	13	4	7	4	3
7	2						
8	5		2	1			
9	1		1	1	1		
>10	873	195	300	224	156	112	111

Requisitions vs Maintenance Factors

Maintenance Factor	Requisition Classes						
	[6,7)	[7,8)	≥ 8				
0	152	120	825				
1	181	152	1098				
2	20	19	210				
3	21	9	149				
4	3		53				
5	12	11	82				
6	4	4	5				
7	1	1	9				
8	—	2	3				
9	1		3				
>10	70	68	880				

Requisitions vs Procurement Lead Time

(months)	Requisition Classes						
PLT	TRIVIAL	[0,1)	[1,2)	[2,3)	[3,4)	[4,5)	[5,6)
[0,3]	163	44	7	4	4	4	
(3,6]	4316	932	742	484	331	200	169
(6,9]	2127	773	712	438	294	205	184
(9,12]	2370	624	556	364	246	182	158
(12,15]	486	168	172	145	114	77	65
(15,18]	169	73	63	60	34	24	24
(18,21]	48	14	18	11	12	8	5
(21,24]	23	5	8	6	3	1	1
(24,27]	3	1		1	2	1	2
(27,30]		1		2			

CHAPTER III

PROBABILITY CHARACTERISTICS OF DEMAND

3.1 Introduction

In this chapter the question of dependency between the zero and non-zero demand states of the series is explored. It was alluded to in the previous chapter by observing the short interarrival times for low demand items that the non-zero demands may occur in clusters. If this is true, then the probability of getting a non-zero demand in the future is dependent on the observed history of the series.

3.2 Leading and Trailing Zeros

One of the problems in analyzing the data was the fact that many of the individual items had clusters of leading or trailing zeros. In the real world these series may represent the phasing in or out of the item and would not be characteristic of the actual demand for the item.

These potential exogeneous observations had no impact on the demand variables* in the previous chapter but were considered in the analysis presented in this chapter.

3.3 The State Series and its Order [8]

Throughout the analysis in this chapter, we employ the state series concept to determine the probability of demand. This concept is as follows:

Let $\{x_t\}$ be the quarterly demand series.

Define $\{y_t\}$ as the state series such that

$$y_t = \begin{cases} 1 \text{ if } x_t > 0 \\ 0 \text{ if } x_t = 0 \end{cases}$$

Define the order of $\{y_t\}$, (the order of a Markov chain), as the smallest k such that y_t is independent of observations prior to y_{t-k} for all t.

---

*These variables were measured between the first and last demands.

17

(i.e. Probability $\{y_t | y_{t-1}, y_{t-2}, \ldots\} = P\{y_t | y_{t-1}, y_{t-2}, \ldots, y_{t-k}\}$)

Hence if k = 0, then

$P(y_t=1)$ or $P(y_t=0)$ is independent of the past

and if k = 1, then

$P\{y_t | y_{t-1} = 0\} \neq P\{y_t | y_{t-1} = 1\}$ and y_t is dependent on the last

quarter of data only. Similarly for all k > 0, the states of y_t are dependent
on the past k observations.

3.4 Determining the Order of the State Series

Sweet in referenced article [8] describes the use of Akaike's information
criterion in determining the order of the state series. In the same article he
includes a program which not only determines the order of the series but also
estimates the transition probabilities. This program was modified so that
several series could be evaluated concurrently and a distribution of the orders
generated. It was hoped that one order would dominate the results.

When testing the above program, it was found that this technique favored
lower orders when the length of the series was relatively small. Several
specific Markov series of length 44 (length of our observations) were tested;
the slightest deviation from the exact series caused the identification of a
lower order and in most cases the order was zero.

When testing the actual series, with and without the leading/trailing zeros,
it came as no surprise that 95% of the series resulted in zero order.

3.5 Wald-Wolfowitz Runs Test

Another method of testing for independence between the 0s and 1s in the state
series is the Wald-Wolfowitz runs test. This is a two tailed test for randomness
using the statistic K, the number of runs in the series. Independence is re-
jected for both large K (too many runs) and small K (too few runs).

It was observed that state series with order 0 had 1s evenly dis-
tributed (not clustered) leading to a large K or rejection of the runs test. On
the other hand, large K did not necessarily imply a non-zero order of the state
series. Hence, even as a one tailed test (large K) the runs test is less con-
servative in rejecting randomness than is the Akaike information approach.

18

Considering the other tail of the test, few runs indicate that the 0s and 1s cluster. This clustering would not be identified in short series by using the Akaike criterion.

The Wald-Wolfowitz runs test was applied to the various sets of data with an alpha level of .05 for a two tailed test. Consistent with the Akaike information finding, randomness was not rejected because of a large K statistic, but on the other hand, there were many cases where K was sufficiently small for rejection, indicating possible clustering. The results of this test are found in Table 1 for the low dollar value dynamic and non-dynamic sets of data.

3.6 Heuristic Approach

In a final effort to determine if dependency is evident in the state series, empirical observations were made on each requisition class of items. Frequency estimates were made for each transition probability for four Markov orders (state series orders), 0 thru 3, using all the series within the requisition class.

Example: Assume order 1, then the transition probabilities in question would be

$$P(0|1)$$
$$P(0|0)$$
$$P(1|1)$$
$$P(1|0)$$

and the point estimate for $P(0|1)$ would be

$$P(0|1) = \frac{NT(1,0)}{NT(1,0) + NT(1,1)}$$

where $NT(1,0)$ = number of times a 1 was followed by a zero in all the series within the requisition class. Similarly $NT(1,1)$ = number of times a 1 was followed by a 1.

Also $P(1|1) = 1 - P(0|1)$

19

Table 1

Results from Wald-Wolfowitz Runs Test

Data Set Regression Class	LDVDYN			LDVNON		
	N	# Dependent	%	N	# Dependent	%
[0,1]	438	133	30	10382	2720	26
1	121	36	30	2311	931	40
2	64	25	39	1349	591	44
3	48	26	54	910	451	50
4	46	25	54	600	325	54
5	24	14	41	513	261	51
6	23	15	65	396	210	53
7	31	14	45	318	172	54
8	334	244	79	2306	1602	69

20

Now if the observations of each series were indeed independent, then the transition probabilities of going to a given state would be the same for all assumed Markov orders. Since clustering appears to be evident, a heuristic to determine if the cluster effect is sufficient to reject the hypothesis of independence is to test:

$$P(0) = P(0|0) = P(0|0,0) = P(0|0,0,0)$$

$$\text{and} \quad P(1) = P(1|1) = P(1|1,1) = P(1|1,1,1)$$

Graphs of the point estimates for each of the above probabilities and for each requisition class are given at the end of the chapter (1160 items). The analysis was done with and without the exongeneous observation (leading and trailing zeros).

The most noteworthy observations from these graphs are as follows:

1. $P(0) < P(0|0) < P(0|0,0) < P(0|0,0,0)$

   $P(1) < P(1|1) < P(1|1,1) < P(1|1,1,1)$

   for all requisition classes.

2. The most significant difference in the above probabilities in all cases are –

$$P(0|0) - P(0) \quad \text{(first order Markov property)}$$
$$\text{and} \quad P(1|1) - P(1)$$

3. The probabilities are requisition class dependent especially at the lower state orders.

4. Omitting the trailing and leading zeros had significant impact on the estimated probabilities.

3.7 Conclusions

The data analysis presented in this chapter did not clearly show any dependent demand pattern in the state series. There was an indication of clustering where zeros tended to follow zeros and similarly ones tended to follow ones. Plots of certain conditional probabilities indicate that this clustering may be best represented by a first order Markov property even though statistical tests did not confirm this. It was also evident from these plots
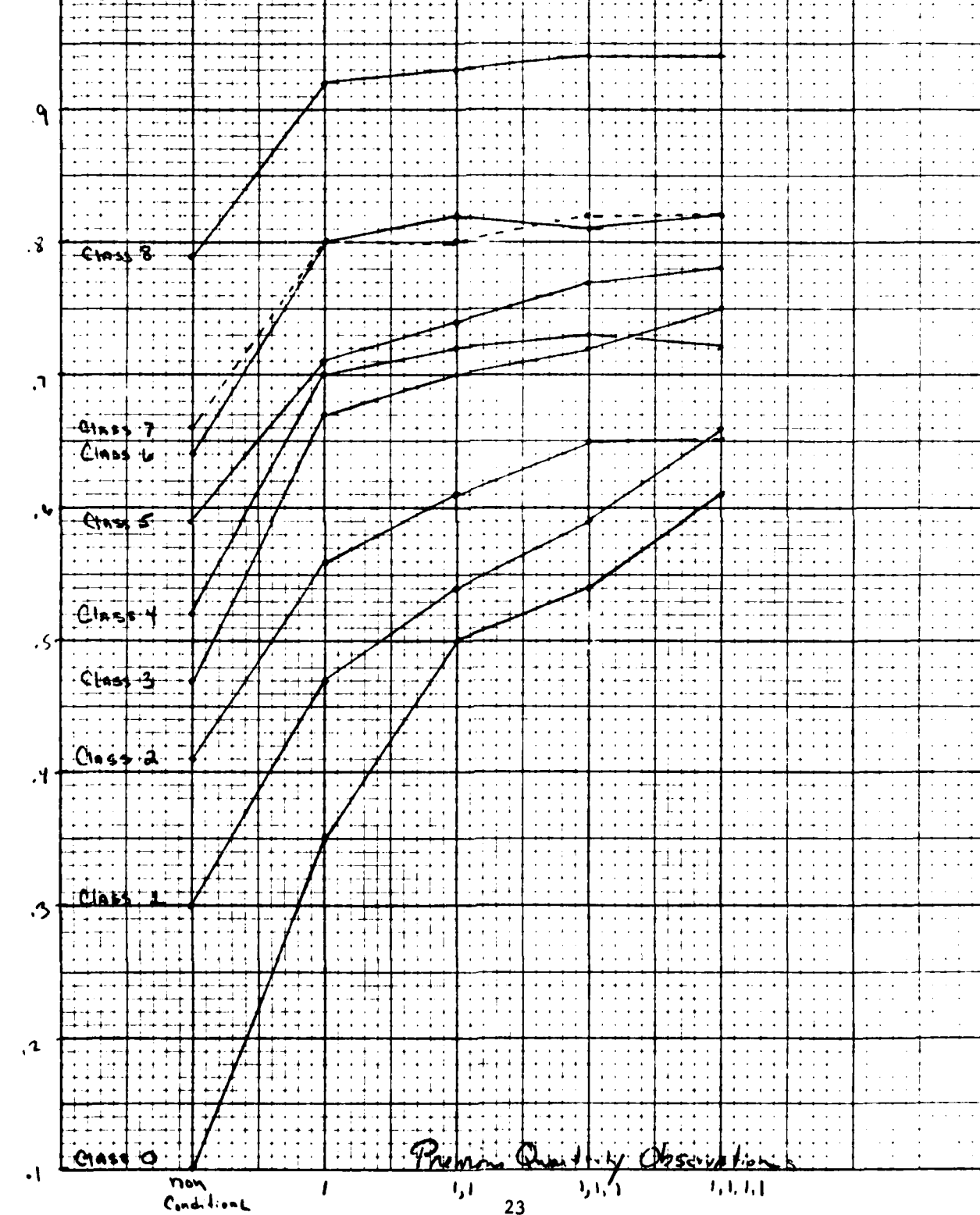
that the probability of demand was dependent on the requisition class and the use of catalog estimates may be useful.

In the next chapter, these observations will be considered assumptions for various models, and based on these assumptions, specific forecast algorithms will be developed.
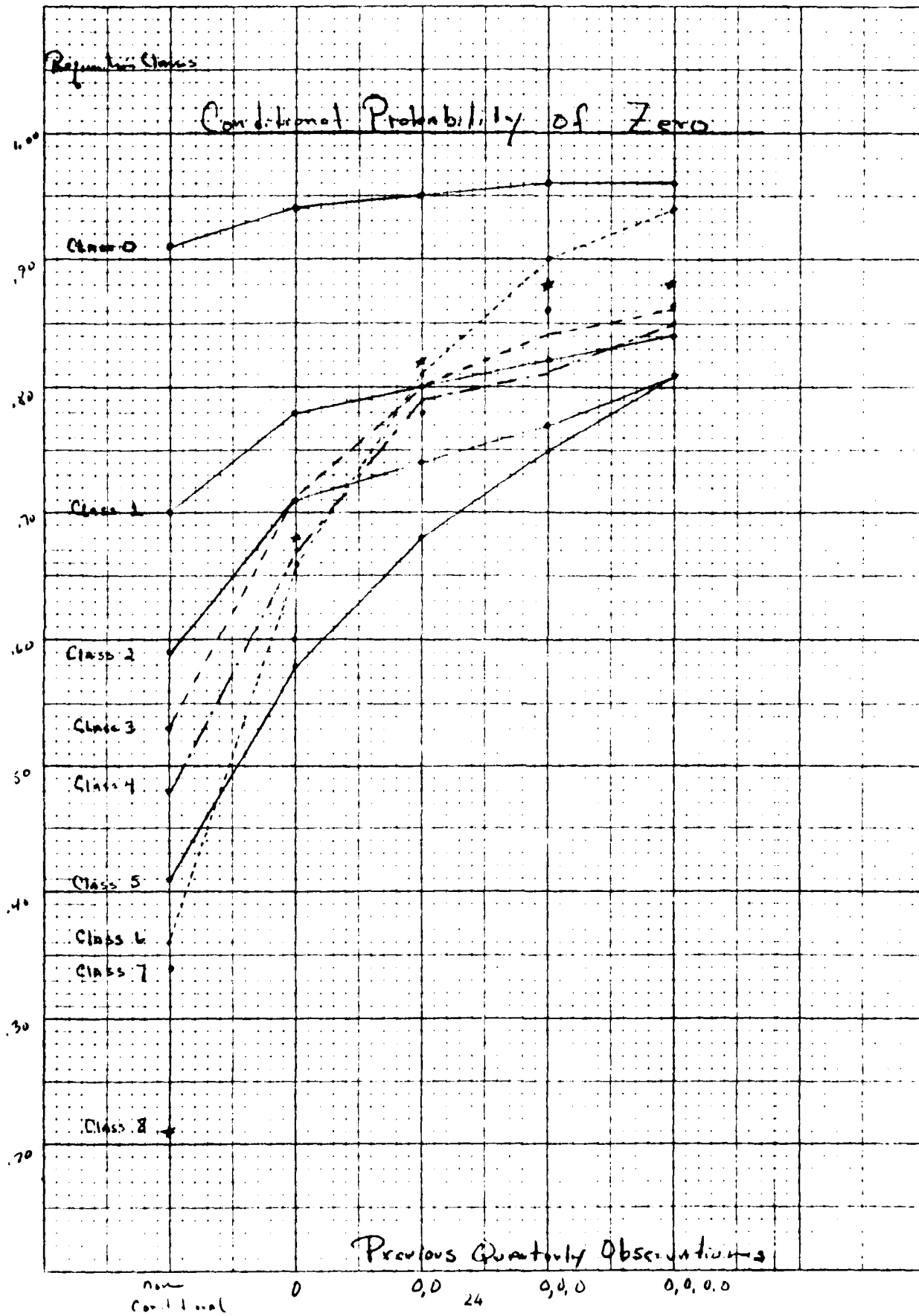
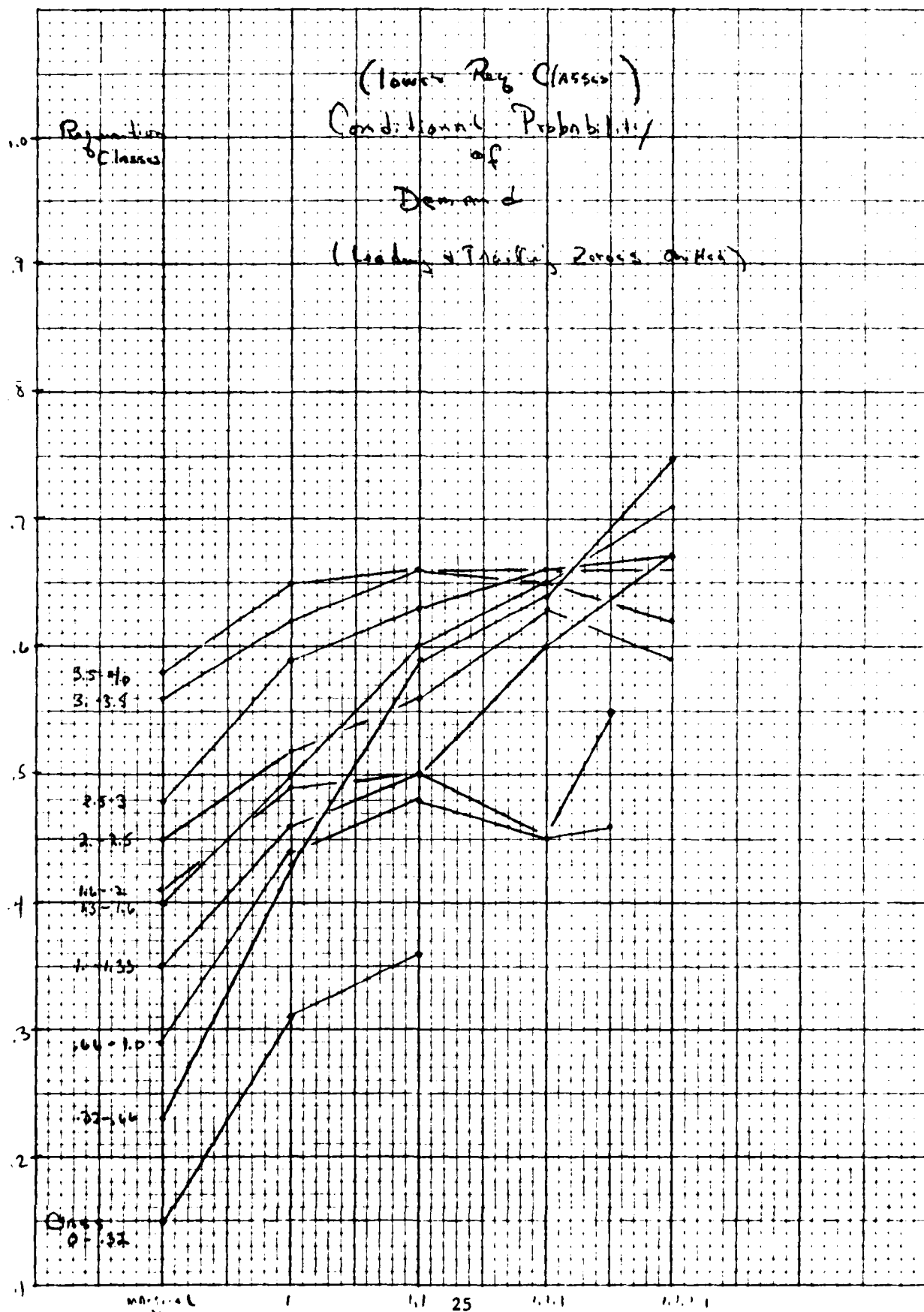LDVDYN - 1169 item...

46 0780

K•E 10 X 10 TO THE INCH...
KEUFFEL & ESSER CO MADE...

Prob

1.0

Requisition Class — Conditional Probability of Demand

.9

.8   Class 8

.7   Class 7
      Class 6

.6   Class 5

.5   Class 4

      Class 3

.4   Class 2

.3   Class 1

.2

.1   Class 0   Previous Quality Observations

      non
      Conditional   1   1,1   1,1,1   1,1,1,1

23

Regression Classes

Conditional Probability of Zero

Class 0

Class 1

Class 2

Class 3

Class 4

Class 5

Class 6

Class 7

Class 8

1.00

.90

.80

.70

.60

.50

.40

.30

.20

Previous Quarterly Observations

non
Conditional

0

0,0

24

0,0,0

0,0,0,0

(lower Reg Classes)
Conditional Probability
of
Demand
(Leading & Trailing Zeroes Omitted)

(lower Req Classes)
Conditional Probability of Zero
(Leading & Trailing Zeros Omitted)

26

Conditional Probability of Zero
No Leading Zeros or Trailing Zeros

Class 0
Class 1
Class 2
Class 3
Class 4
Class 5
Class 7
Class 8

Conditional Probability of Decision
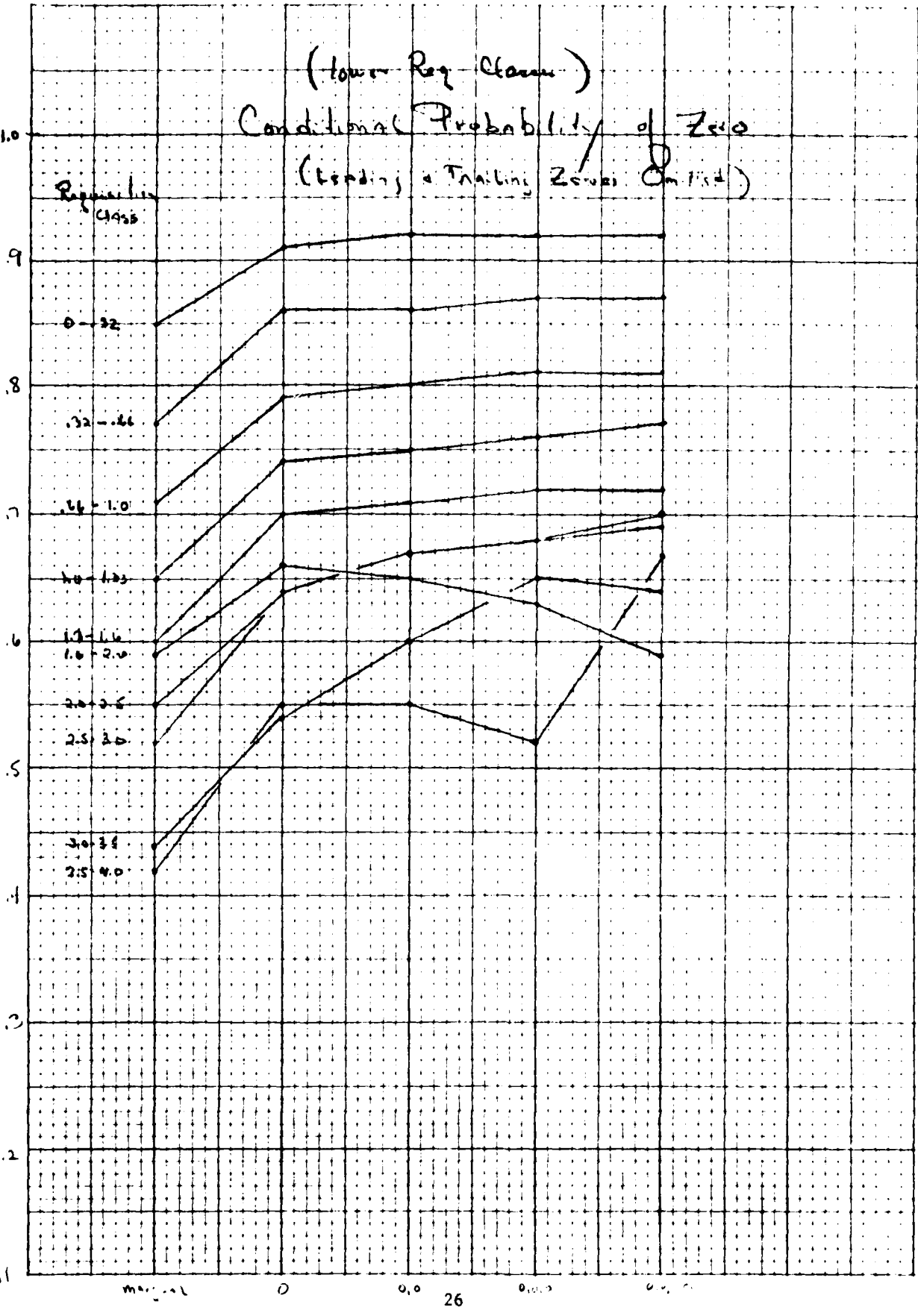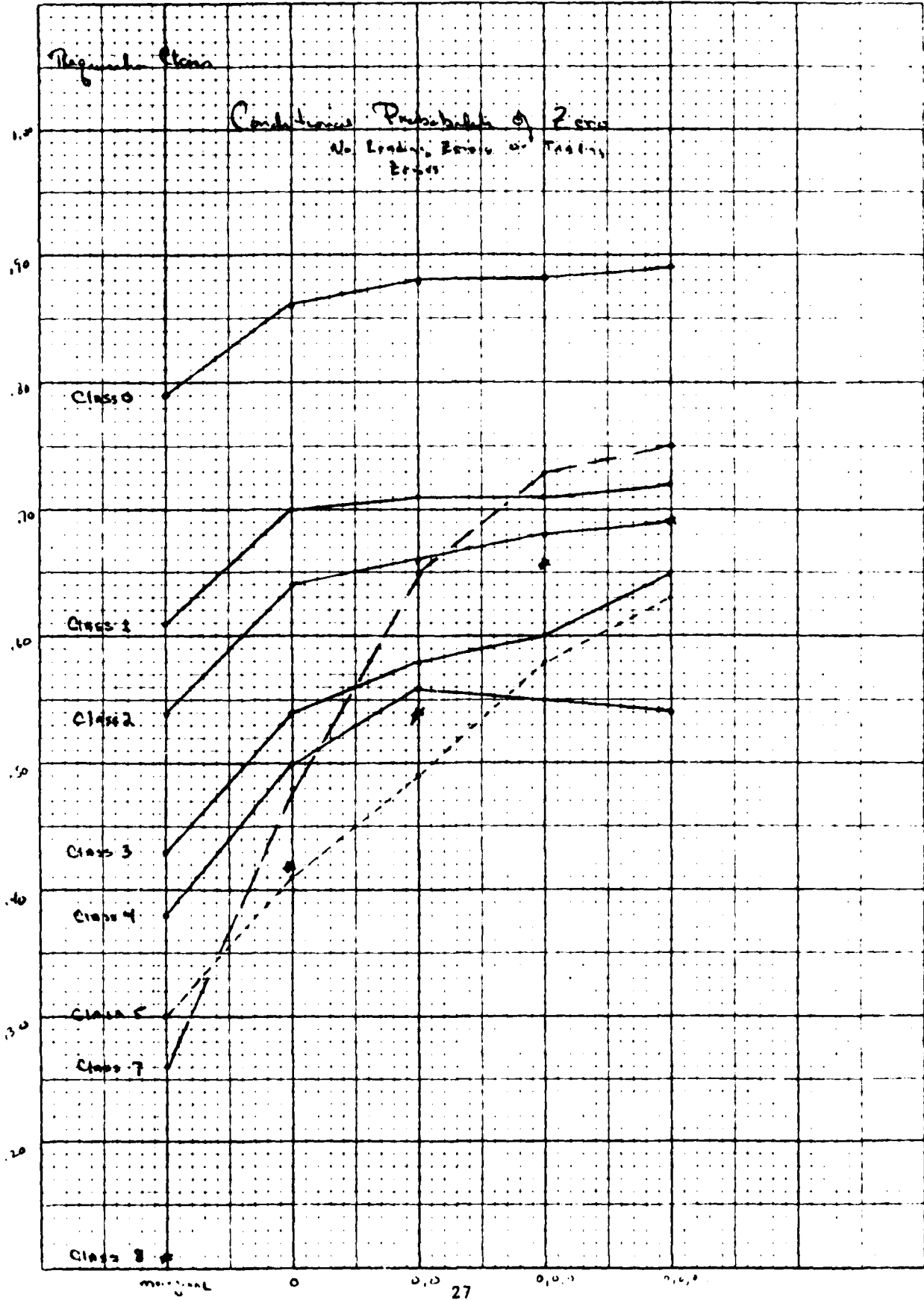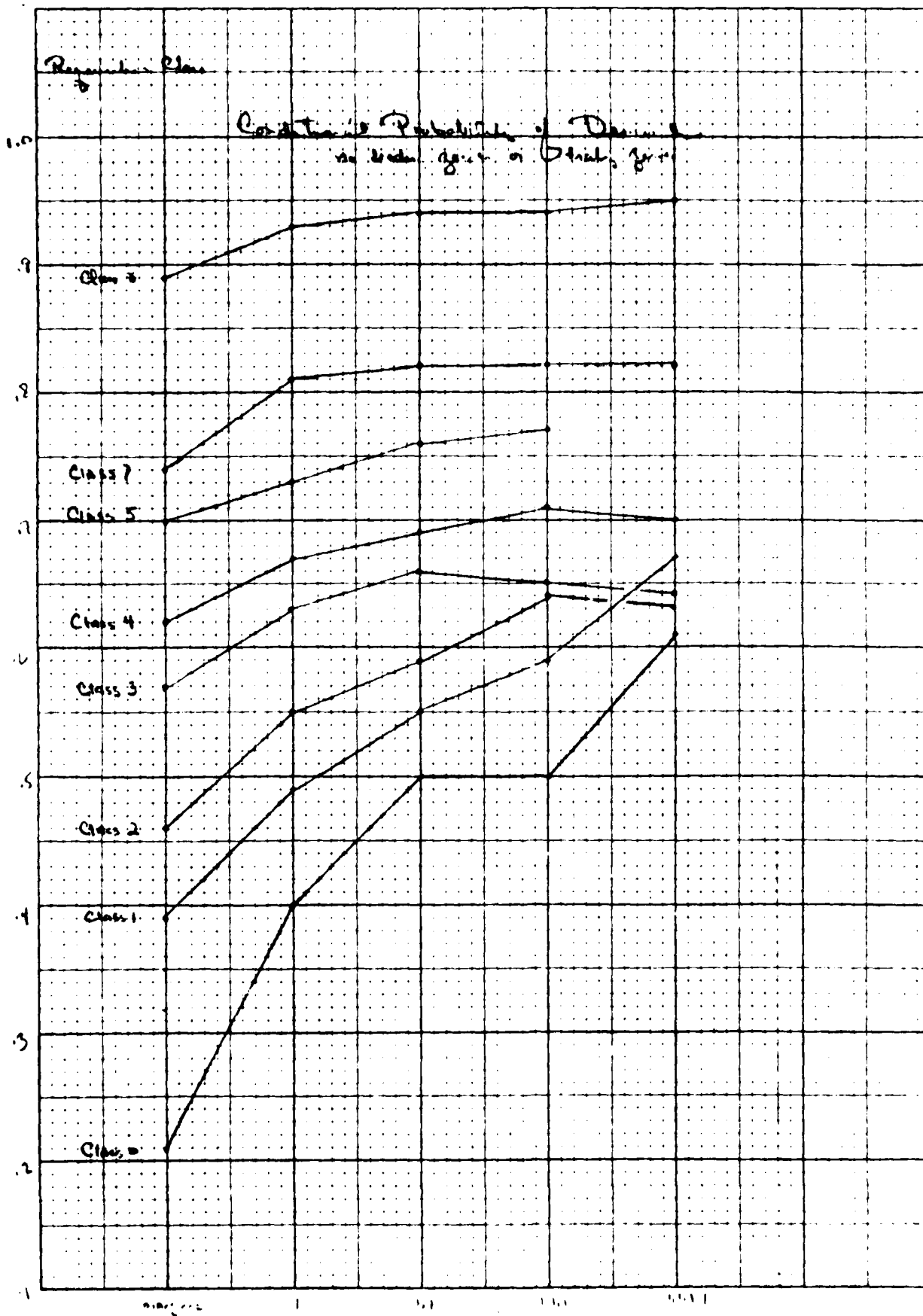
CHAPTER IV

FORECAST ALGORITHM DEVELOPMENT

4.1 Background

For low demand or inactive items, both Croston [5] and Sweet [8] recommended making separate estimates of demand size and the probability of getting a demand. Croston pointed out that in an inventory environment, traditional forecast techniques such as exponential smoothing on the demand series often resulted in excess stock. Sweet demonstrated, using weekly demands for rolled steel, that this decomposition approach was superior to traditional methods.

These techniques differed only in their way of estimating the probability of getting a future demand. Croston modelled the interarrival times of demand and developed an appropriate estimate whereas Sweet modelled the Markov properties of the state series and estimated the transition probabilities once the order of the state series was established. Both methods relied on traditional techniques to forecast the conditional size of demand given a demand occurred.

In the sections that follow, both Croston's and Sweet's methods are used to develop expected value forecast algorithms based on model assumptions that appear appropriate from the empirical observations cited in the previous chapter. These techniques are adapted to inventory models where a forecast for the total demand over an item's procurement lead time (PLT) is needed. This PLT is variable between items.

4.2 Notation

Let $\{x_t\}$ be the quarterly demand series of a given item, $t = 1,2,3,\ldots$ and let

PLT be the item's procurement lead time in quarters

Then define the following series:

State Series

$\{y_t\}$ such that $y_t = \begin{cases} 1 \text{ if } x_t > 0 \\ 0 \text{ if } x_t = 0 \end{cases}$

Lead Time State Series

$$\{z_t\} \quad \text{where } z_t = \sum_{j=1}^{PLT} y_{t+j}$$

$$z_t = 0,1,2,\ldots PLT$$

Non-Zero Demand Series

$$\{d_j\} \quad \text{where } d_j = j^{th} \text{ non-zero demand in series } \{x_t\}$$

Note: At time t, let

$$N(t) = \max \{j \mid d_j \text{ occurred by time } t\}$$

then

$$d_{N(t)} = \text{last non-zero demand observed by time } t$$

$$d_{N(t)} + \ell = \ell^{th} \text{ non-zero demand occurring after time } t$$

Interarrival Time Series

$$\{I_j\} \quad \text{where } I_j = \text{time in quarters between } d_j \text{ and } d_{j-1}$$
$$\text{of the original series}$$

Note: At time t

$$I_{N(t)} = \text{last interarrival time observed by time } t$$

$$I_{N(t)+\ell} = \ell^{th} \text{ interarrival time after time } t$$

Composite Series

$$\{x_t\} = \{y_t \cdot d_{N(t)}\} \qquad t = 0,1,2,\ldots$$

4.3 Independent Model

Assumptions

a. $\{y_t\}$ is a sequence of Bernoulli trials

where $P(y_t = 1) = p$ and

$$P(y_t = 0) = 1 - p = q$$

for every t

b. $\{d_j\}$ is independent of $\{y_t\}$ and $\hat{d}_{j+\ell}$ = traditional estimate

of $d_{j+\ell}$ having observed

$d_1, d_2, \ldots, d_j$

Forecast Development

Now by using the $\{z_t\}$ series (the number of non-zero demands during a PLT period after time t) and the fact that $\{y_t\}$ is independent of $\{d_j\}$, we have the expected demand for a given $\{d_j\}$ series after time t for a PLT period is:

$$E\left[\sum_{i=1}^{PLT} x_{t+i}\right] = \sum_{i=1}^{PLT}\left[P(z_t = i)\sum_{\ell=1}^{i}(d_{N(t)+\ell})\right]$$

$$= \sum_{i=1}^{PLT}\left[\binom{PLT}{i} p^i q^{PLT-i}\sum_{\ell=1}^{i}(d_{N(t)+\ell})\right]$$

where

$P(z_t = i)$ = binomial probability of exactly i non-zero demands in PLT periods

and if the non-zero demand series is stationary

i.e. $(d_{N(t)+\ell}) = (d + (\text{random noise}))$ for every PLT

then $E\left[\sum_{i=1}^{PLT} x_{t+i}\right] = \sum_{i=1}^{PLT} P(z_t = i)(i)(d)$

$$= (PLT)(p)(d)$$

Now the PLT forecast at time t is

$$\sum_{i=1}^{PLT} \hat{x}_{t+i} = \sum_{i=1}^{PLT}\left[\binom{PLT}{i}(\hat{p}_t)^i (1-\hat{p}_t)^{PLT-i}\sum_{\ell=1}^{i}\hat{d}_{N(t)+\ell}\right]$$

or if $\{d_j\}$ is a stationary series

$$\sum_{i=1}^{PLT} \hat{x}_{t+i} = (PLT)(\hat{p}_t)\,\hat{d}_{N(t)+1}$$

where $\hat{p}_t$ is an estimate from $y_1, y_2, \ldots, y_t$ or a catalog of similar series (e.g. based on requisition classes)

and $\hat{d}_{N(t)+1}$ is an estimate from $d_1, d_2, \ldots, d_{N(t)}$

31

4.4 First Order Dependence Model

Assumptions

a. $\{y_t\}$ is a first order Markov process, i.e. $P(y_t|y_{t-1}, y_{t-2},\ldots,y_{t-k}) = P(y_t|y_{t-1})$ for every t and k, $0 < k < t$

define $P(y_t = j|y_{t-1} = i) = P_{ij}$ for i and j either 0 or 1

b. Same as b from Independent Model

Forecast Algorithm Development

The expected lead time demand is, as before,

$$E\left[\sum_{i=1}^{PLT} x_{t+i}\right] = \sum_{i=1}^{PLT}\left[P(z_t=i)\sum_{\ell=1}^{i}(d_{N(t)+\ell})\right]$$

But now $P(z_t = i)$ is more complex due to the first order dependency of the $\{y_t\}$ series. To compute $P(z_t = i)$ consider the following argument:

Let $\Omega = \left\{(y_{t+1}, y_{t+2},\ldots,y_{t+PLT})\mid y_{t+k} = 0,1\right\}$

be the set of all possible future events or outcomes of the state series.

Then there exist exactly $\binom{PLT}{i} = \dfrac{PLT!}{i!\ (PLT-i)!}$

events that satisfy $z_t = i$ (the number of ways of arranging i ones and PLT-i zeros).

Now let $\delta = \left\{E_{11}, E_{12}, E_{13},\ldots E_{1\binom{PLT}{i}}\right\}$ be the subset of events satisfying $z_t=i$.

Then $P(z_t=i) = \sum_{j=1}^{\binom{PLT}{i}} P(E_{ij}|y_t)$ (Markov Property)

where $P(E_{ij}|y_t)$ is computed using the transition probabilities as illustrated by the example at the end of this section.

32

Now $E\left[\sum_{i=1}^{PLT} x_{t+i}\right] = \sum_{i=1}^{PLT}\left[\sum_{j=1}^{\binom{PLT}{i}} P(E_{ij}|y_t)\sum_{\ell=1}^{i}(d_{N(t)+\ell})\right]$

and the PLT forecast at time t is

$$\sum_{i=1}^{PLT}\hat{x}_{t+i} = \sum_{i=1}^{PLT}\left[\sum_{j=1}^{\binom{PLT}{i}}\hat{P}(E_{ij}|y_t)\sum_{\ell=1}^{i}\hat{d}_{N(t)+\ell}\right]$$

or if $\left\{d_j\right\}$ is a stationary series

$$\sum_{i=1}^{PLT} x_{t+i} = \hat{d}_{N(t)+1}\sum_{i=1}^{PLT} i\sum_{j=1}^{\binom{PLT}{i}}\hat{P}(E_{ij}|y_t)$$

where $\hat{P}(E_{ij}|y_t)$ is computed using $\hat{P}_{00}$, $\hat{P}_{01}$, $\hat{P}_{11}$, $\hat{P}_{10}$ which are estimates from y_1, $y_2,\ldots,y_t$ or from a catalog of similar series and $\hat{d}_{N(t)+\ell}$ is estimated from d_1, $d_2,\ldots,d_{N(t)}$ as in the independent model.

<u>Example</u>: Suppose PLT = 4, i = 2. Then there exist exactly $\frac{4!}{2!2!}$ events satisfying the condition $z_t = i = 2$, one of these being $E_{21} = (0,0,1,1)$

where $P(E_{21}|y_t = 0) = P_{00} \cdot P_{00} \cdot P_{01} \cdot P_{11}$

and $P(E_{21}|y_t = 1) = P_{10} \cdot P_{00} \cdot P_{01} \cdot P_{11}$

4.5 <u>Interarrival Time Model</u>

<u>Assumptions</u>

a. $\left\{I_j\right\}$ is a sequence of independent exponentially distributed random variables such that

$$P(I_j = t) = \lambda^{-1}\exp(-t/\lambda), \quad 0 < t < \infty$$

b. Same as b from independent model.

<u>Forecast Algorithm Development</u>

The expected lead time demand is, again:

$$E\left[\sum_{i=1}^{PLT} x_{t+i}\right] = \sum_{i=1}^{PLT}\left[P(z_t = i)\sum_{\ell=1}^{i} d_{N(t)+\ell}\right]$$

33

Now $P(z_t = i)$ is the probability of getting i demand arrivals in a PLT (time), and z_t is a Poisson process with

$$P(z_t = i) = \frac{e^{-\lambda PLT}(\lambda \cdot PLT)^i}{i!} \quad i = 0,1,2,\ldots$$

and

$$E\left[\sum_{i=1}^{PLT} x_{t+1}\right] = \sum_{i=1}^{PLT}\left(\frac{(e^{-\lambda \cdot PLT})(\lambda \cdot PLT)^i}{i!} \sum_{\ell=1}^{i} d_{N(t)+\ell}\right)$$

Now the PLT forecast at time t is

$$\sum_{i=1}^{PLT} \hat{x}_{t+1} = \sum_{i=1}^{PLT} \frac{e^{-\hat{\lambda} \cdot PLT}(\hat{\lambda} \cdot PLT)^i}{i!} \sum_{\ell=1}^{i} \hat{d}_{N(t)+\ell}$$

or if $\left\{d_j\right\}$ is a stationary series

$$\sum_{i=1}^{PLT} \hat{x}_{t+1} = \hat{d}_{N(t)+1} \sum_{i=1}^{PLT} \frac{i \cdot e^{-\hat{\lambda} \cdot PLT}(\hat{\lambda} \cdot PLT)^i}{i!}$$

$$= \left(d_{N(t)+1} \cdot \hat{\lambda} \cdot PLT\right) - \sum_{i=PLT+1}^{\infty} \frac{e^{-\lambda \cdot PLT}(\hat{\lambda} \cdot PLT)^i \cdot i}{i!}$$

$$\approx \hat{d}_{N(t)+1}\, \hat{\lambda} \cdot PLT$$

where $\frac{1}{\lambda}$ is an estimate of the mean of the interarrival series $I_2, I_3,\ldots,I_{N(t)}$ or the mean of the interarrival times from a catalog of similar series and $\hat{d}_{N(t)+\ell}$ is estimated from $d_1, d_2,\ldots,d_{N(t)}$

The following are comments on the assumptions:

    a. I_j is an integer variable where the interarrival time is measured in quarters (not continuous as model assumes).

    b. Z_t is truncated in that $Z_t = 0,1,2,\ldots PLT$.

The implications of these contradictions to the model have not been fully identified but it appears that the forecast algorithm may be reasonable based on experience cited by Croston and the fact that the resulting algorithm is not that different

from the Independent Model.

4.6 Conclusions

The three forecast algorithms developed in this chapter were derived so
that they would be unbiased estimates of the mean of future demand over a PLT
for each specific model. In no case was consideration given to minimizing
any error criterion such as mean squared error. In the next chapter, the
problems with evaluation in an inventory environment will be considered, par-
ticularly for the low demand series considered in this report.

CHAPTER V

EVALUATION

5.1 Background

There is no clearly defined methodology for evaluating forecast algorithms in an inventory management system. In the past, ref [11], statistical error measures such as mean squared error (MSE), mean absolute deviation (MAD) and functions thereof have yielded inconsistent rankings among themselves and with the current IRO supply simulator. Also findings from [6] and the table in the Appendix illustrate that for series with many zero observations the traditional statistical error measures favor algorithms yielding routinely small forecasts and in fact will probably most favor the degenerate algorithm which forecasts zero all the time. This, of course, is not necessarily an optimal policy for a supply system.

As an alternative, it may be proper in a philosophical sense to investigate forecast algorithms as they interrelate with a suitable supply management policy for inactive items. Since we have plans to analyze modified supply policies, the rankings of algorithms under a simulator of current policy are moot. For example a reasonable policy may be to order when the probability of a demand next period is highest, in which case an algorithm which accurately projects probabilities of any demand in near term would be valuable.

For these reasons a statistical surrogate of a simulator's cost-performance measure was utilized in the current analysis of inactive items. Details of the development and experience with this evaluation method can be found in [11].

5.2 Methodology

Overview

The impact of an overforecast error in predicting demand is different from that of an underforecast. Overforecasts result in carrying too much stock which imposes a higher cost to the system whereas underforecasts result in requisitions not being satisfied and a degradation in performance. It is this tradeoff between these two types of errors which is the impetus behind the evaluation methodology described in this section.

This cost-vs-performance relationship is attained by computing two error measures for each item and rolling the measures up over the items. The cost measure computed each time an overforecast is made is an estimate

36

of the percent of total dollar demand which is spent on extra stock.
Similarly the performance measure computed each time an underforecast is
made is an estimate of the total number of requisitions not satisfied.
(Refer to the section on error measures for details.)

For each forecast algorithm tested, various performance levels are
computed by adjusting the forecast (adding or subtracting a percent of the
forecast) to represent the impact of adding a safety level (negative or
positive). With each level of performance a corresponding cost is computed
yielding a cost vs performance curve for each forecast method.
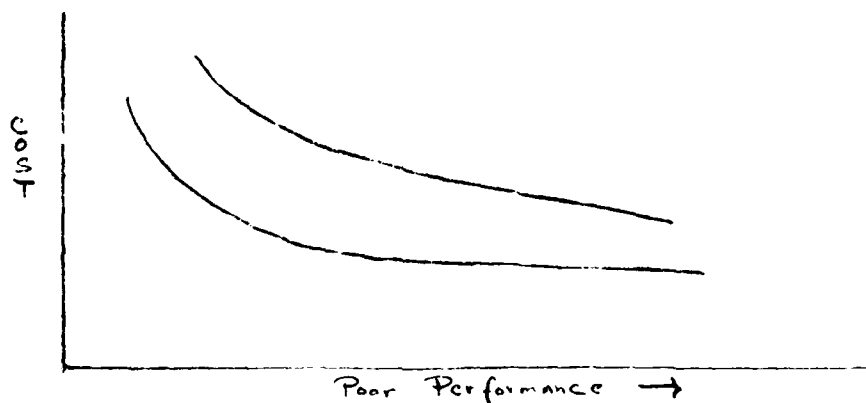
Curves closest to the x-y axis represent better performance or lower cost.

Error Measures

Notation

For the given i^{th} demand series $\{x_{ij}\}$ and its corresponding $\{r_{ij}\}$
requisition series (the number of requisitions in period j)

Let $D_{i,t}(\ell) = \sum_{j=1}^{\ell} x_{i,t+j}$ be the demands over ℓ periods from time t

$\widehat{D}_{i,t}(\ell)$ = one of the lead time forecasts from Chapter IV.

$R_{i,t}(\ell) = \sum_{j=1}^{\ell} r_{i,t+j}$ be the number of requisitions over ℓ periods from time t

$EL_{i,t}(\ell) = (\widehat{D}_{i,t}(\ell) - D_{i,t}(\ell)) = \sum_{j=1}^{\ell} (x_{i,t+j})$ be the errors
over lead time ℓ made at time t

37

UP_i = unit price of the ith item

F_i = index set of forecasts for i^{th} item, i.e. those times at which forecasts are needed.

Cost (Overforecost) Measure

$$OF(i) = \sum_{j \in F_i} \max\left[\frac{EL_{i,j}(8)}{D_{i,j}(8)}, 0\right] D_{i,j}(8) \; UP_i$$

$$= \sum_{j \in F_i} \max\left[EL_{i,j}(8), 0\right] UP_i$$

$$OF = \frac{\sum_{i=1}^{N} OF(i)}{\sum_{i=1}^{N} UP_i \sum_{j \in F_i} D_{i,j}(8)}$$

is a percent of the total dollar demand spent on extra stock. The base period 8 is used to capture the long term effect of an overforecast.

Performance (Underforecast) Measure

$$UF(i) = \sum_{j \in F_i} \max\left[-\frac{EL_{ij}(PLT)}{D_{ij}(PLT)}, 0\right] R_{ij}(PLT)$$

is an estimate of the number of requisitions not satisfied for the ith item, i.e. if demand is underforecasted, say by 20%, then it is implied that 20% of the requisitions will not be satisfied.

$$UF = \frac{\sum_{i=1}^{N} UF(i)}{\sum_{i=1}^{N} \sum_{j \in F} (R_{ij}(PLT))}$$

is an estimate of the percent of the total requisitions not satisfied over all the items.

The base period is shortened to the procurement lead time, which represents the quickest time stock could be replenished after a new order is placed. In an underforecast situation the reorder point will probably be crossed within a PLT.

Data Collection Procedures

The following describes the method for collecting the statistical data from the actual series:

(1) Each series starts with the first non-zero demand. (leading zeros are omitted)

(2) Forecasts are made only after the 8th quarter.

(3) Forecasts are made only in a quarter in which a demand has occurred.

(4) The statistics are collected after the 13th quarter.

(5) The forecasts are rounded to the next highest integer.

(6) Forecasts are made up to PLT periods before the end of the series.

(7) Each item's PLT is used when computing the underforecast measure.

5.3 Algorithm Implementation

Assuming that the non-zero demand series $\{d_j\}$ is stationary (no trends), the forecast from each algorithm developed in Chapter IV may be expressed as an estimate of the mean number of non-zero demand quarters in the next PLT quarters times an estimate of the mean demand in a quarter given that there is a demand, i.e.,

$$\sum_{J+L}^{PLT} \hat{X}_{T+j} = \widehat{E[z_t]} \cdot \widehat{E[d_{N(t)+1}]} = \widehat{E[z_t]} \, \hat{d}$$

The $\{d_j\}$ series (non zero demands) is assumed independent of the $\{z_t\}$ series for each model and consequently its mean is estimated the same for each algorithm via a single average of all previous observations of the $\{d_j\}$ series, i.e.

$$\hat{d} = \hat{d}_{N(t)+1} = \sum_{i=1}^{N(t)} d_i / N(t)$$

It was felt that since the actual series were so short (28 periods) and the resulting series $\{d_j\}$ would be even shorter, there would be little to gain in using other procedures such as exponential smoothing to estimate d.

The expected number of non-zero demand quarters in the next PLT quarter is model dependent and is computed by using a catalog of estimated model parameters. These catalogs are broken out by the requisition class of the series. and/or the length of the PLT. The estimated parameters were computed using the total data base, which was stratified by requisition class.

The basic steps in computing the forecast for each algorithm is as follows:

Step 1: Identify the requisition class of the series (compute the average number of requisitions per year for the previous two years - this allows the series to migrate between classes over time).

Step 2: Using the item's PLT, the results from Step 1, and the model's catalog estimate the expected number of non-zero demand quarters $(E(Z_t))$ in the next PLT quarters. (Refer to the next 3 paragraphs.)

Step 3: Compute the average number of demands of the non-zero demand quarters from the item's previous history.

Step 4: Multiply the results from Step 2 and Step 3.

The catalogs as computed for each model are as follows:

Independent Model

For the independent model it was shown that z_t has a binomial distribution with parameter p (Probability of a non-zero demand quarter) and

$$E[z_t] = PLT \times p \qquad z_t = 0,1,2,\ldots,PLT$$

The catalog for this model contains the estimate of p for each requisition class. These are as follows:

Independent Model

REQ Class	[0,.32]	(32,.66]	(.66,1]	(1,1.33]	(1.33,1.6]	(1.6,2]	(2,3]	(3,4]	(4,5]	(5,6]	(6,7]	(7,8]
$\hat{p}$	.15	.23	.27	.35	.40	.41	.46	.57	.62	.7	.74	.89

Dependent Model

For the dependent model the expected value of z_t depends on transition probabilities p_{01} and p_{11} in the following manner.

$$E[z_t] = \sum_{i=1}^{PLT} \sum_{j=1}^{\binom{PLT}{i}} p(E_{ij}|y_t) \qquad (i) \qquad \text{(refer to page 33)}$$

where $p(E_{ij}|y_t)$ can be computed directly from p_{01} and p_{11}. Hence the only parameters that need be estimated are p_{01} and p_{11}. Also $E[z_t]$ depends on the PLT through a series of iterations over $\binom{PLT}{i}$ events which does not have a simplified expression. A catalog was generated using the estimates of p_{01} and p_{11} for each requisition class and yielding $E[z_t|PLT$ and Req Class]. The catalog is shown on page 41.

DEPENDENT MODEL

$$(E(z_t \mid PLT, REQ\ CLASS))$$

PLT \ REQ CLASS	[0,.32]	(.32,.66]	(.66,1]	(1,1.33]	(1.33,1.6]	(1.6,2]	(2,2.5]	(2.5,3]	(3,3.5]	(3.5,4]	(4,5]	(5,6]	(6,7]	(7,8]
1	.31	.44	.45	.46	.49	.50	.52	.59	.62	.65	.73	.77	.81	.93
2	.47	.71	.77	.81	.88	.92	.96	1.10	1.17	1.23	1.40	1.48	1.56	1.83
3	.59	.93	1.05	1.14	1.25	1.33	1.39	1.6	1.72	1.8	2.05	2.17	2.30	2.73
4	.71	1.14	1.33	1.47	1.62	1.73	1.82	2.09	2.26	2.37	2.7	2.9	3.1	3.6
5	.83	1.34	1.61	1.79	2.00	2.14	2.25	2.59	2.80	2.94	3.36	3.53	3.7	4.5
6	.94	1.54	1.89	2.12	2.37	2.54	2.68	3.08	3.34	3.51	4.02	4.30	4.50	5.40
7	1.06	1.74	2.16	2.44	2.74	2.94	3.11	3.57	3.88	4.07	4.7	5.00	5.23	6.30
8	1.17	1.94	2.43	2.81	3.11	3.35	3.53	4.06	4.43	4.64	5.33	5.6	5.9	7.2

41

Interarrival Time Model

For the interarrival time model $\{z_t\}$ is assumed to have a Poisson distribution with

$$E(z_t) = \lambda \cdot PLT \qquad\qquad z_t = 0,1,2,\ldots$$

The only estimate in this case is $\hat{\lambda}$ or $\frac{1}{\hat{\lambda}}$ which is estimated from the interarrival time series $\{I_t\}$ of similar requisition classes. The resulting catalog is as follows:

Interarrival Time Model

REQ	(0,.5]	(.5,1]	(1,1.5]	(1.5,2]	(2,2.5]	(2.5,3]	(3,3.5]	(3.5,4]	(4,4.5]	(4.5,5]	(5,6]
$\hat{\lambda}$	11	5.06	3.2	2.7	2.3	1.9	1.8	1.4	1.3	1.2	1.1

5.4 Results

Experimental Design: Two samples were used for the evaluation. The first and smallest sample consisted of 230 inactive items and was used to determine the best method from each of the *following two groups*.

Decomposition Group

IND = Independent Model - Page 30

DEP = Dependent Model - Page 32

INT = Interarrival Time Model - Page 33

Control Group

1794 (Method currently used) = eight-quarter sum of the demands divided by eight quarter sum of the flying hours.

MOVD = eight quarter moving average on demands only.

The second sample consisted of 9228 items and was used for the final evaluation. This sample was stratified into nine classes based on yearly dollar demand and requisitions per year. The following table lists the item count and total dollar demand for each cell. Classes 1,2,4, and 5 were used for the analysis.

ITEM COUNT AND YEARLY DOLLAR DEMAND

YEARLY $ DEMAND / YEARLY # OF REQ	0 - $5000	$5000 - $50000	$50000
	Class 1	Class 2	Class 3
0-3	# Items = 5795 Total $ DEMAND = $3,872,620	# = 98 $ = 4,154,851	No Sample
3^+ - 12	Class 4 # = 1943 $ = 14,690,818	Class 5 # = 231 $ = 33,001,728	Class 6 No Sample
> 12	Class 7 # = 789 $ = 18,803,075	Class 8 # = 363 $ = 82,521,146	Class 9 # = 6 $5,843,061

A separate Cost vs Performance curve was computed for each algorithm by adjusting the forecast by each of four multiplitive factors (.5,1,1.6,2). The resulting curves were analyzed by comparing performance for a fixed standard cost. (1794 cost at 15% of requisitions not satisfied representing cost currently being incurred at current performance)

Inferences from the Graphs (Refer to pages 45 thru 49) are as follows:

(Performance is in terms of requisitions not satisfied by the end of a lead time)

    (1) Graph 1 (Small Sample)

        (a) Dependent Model satisfied an additional 5.5% of total requisitions over 1794 for the same fixed cost.

        (b) Interarrival Time Model performed poorly compared to the other decomposition models.

        (c) 1794 and the Dependent model were chosen for the final evaluations.
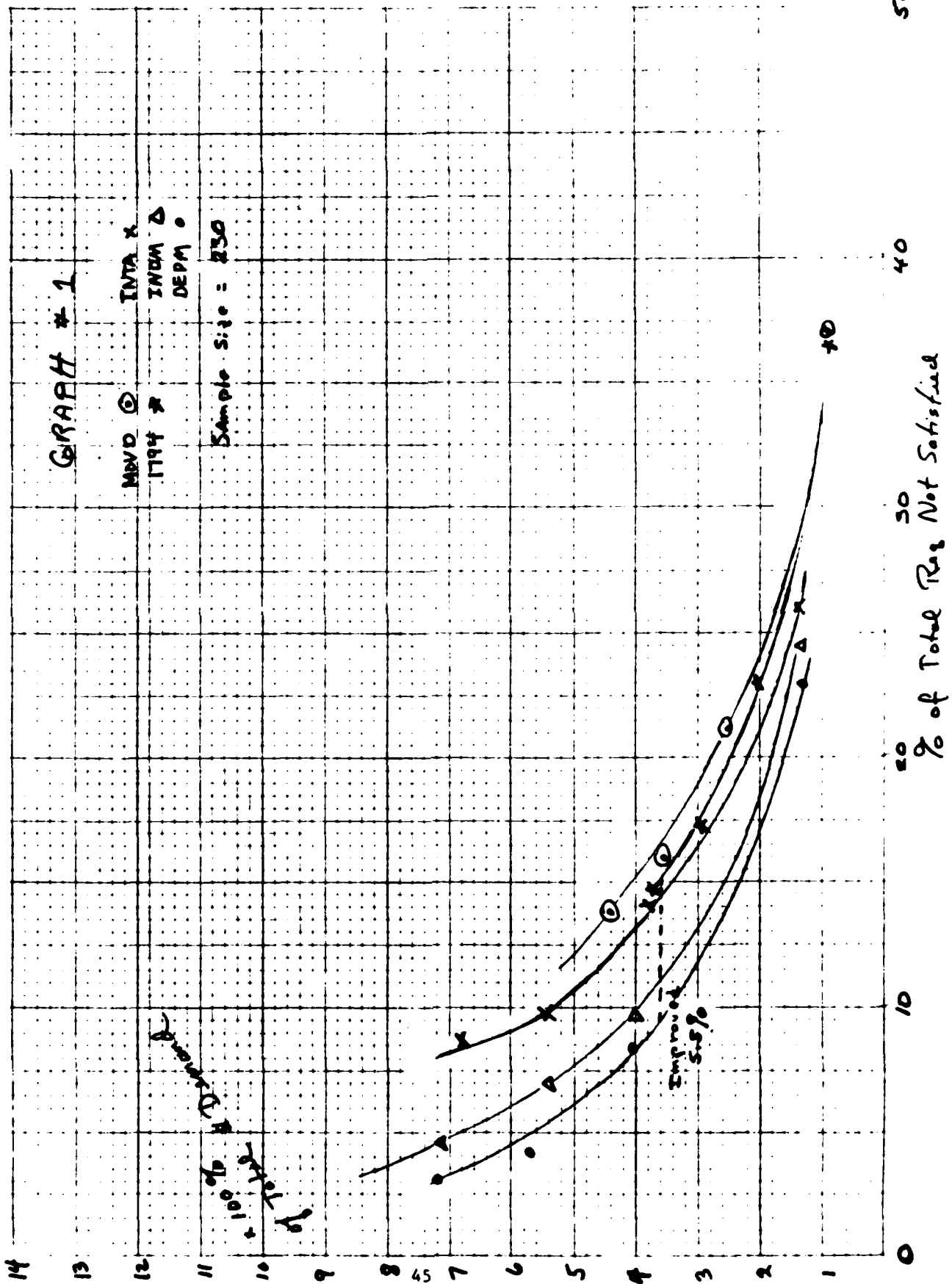
    (2) Graph 2 (Strat Class 1)

        (a) This class contains most of the items but with the least total dollar demand. (See Table on page 43)

        (b) Dependent Model satisfied an additional 6.5% of total requisitions over 1794 for the same fixed cost.

        (c) The curves on this graph are very similar to those from first sample.

    (3) Graph 3 (Strat Class 2)

        (a) This class has higher dollar demand activity.

        (b) Dependent Model satisfied an additional 2.5% requisitions over 1794 for the same fixed cost.

    (4) Graphs 4 and 5 (Strat Class 4 and 5)

        (a) These classes represent greater requisition activity than the previous.

        (b) It is clear that there is no advantage to using the Dependent Algorithm for these classes of items.

        (c) The Dependent Algorithm converges to a long term average as the requisition activity increases. This type of algorithm has not worked well in previous studies with the active items.
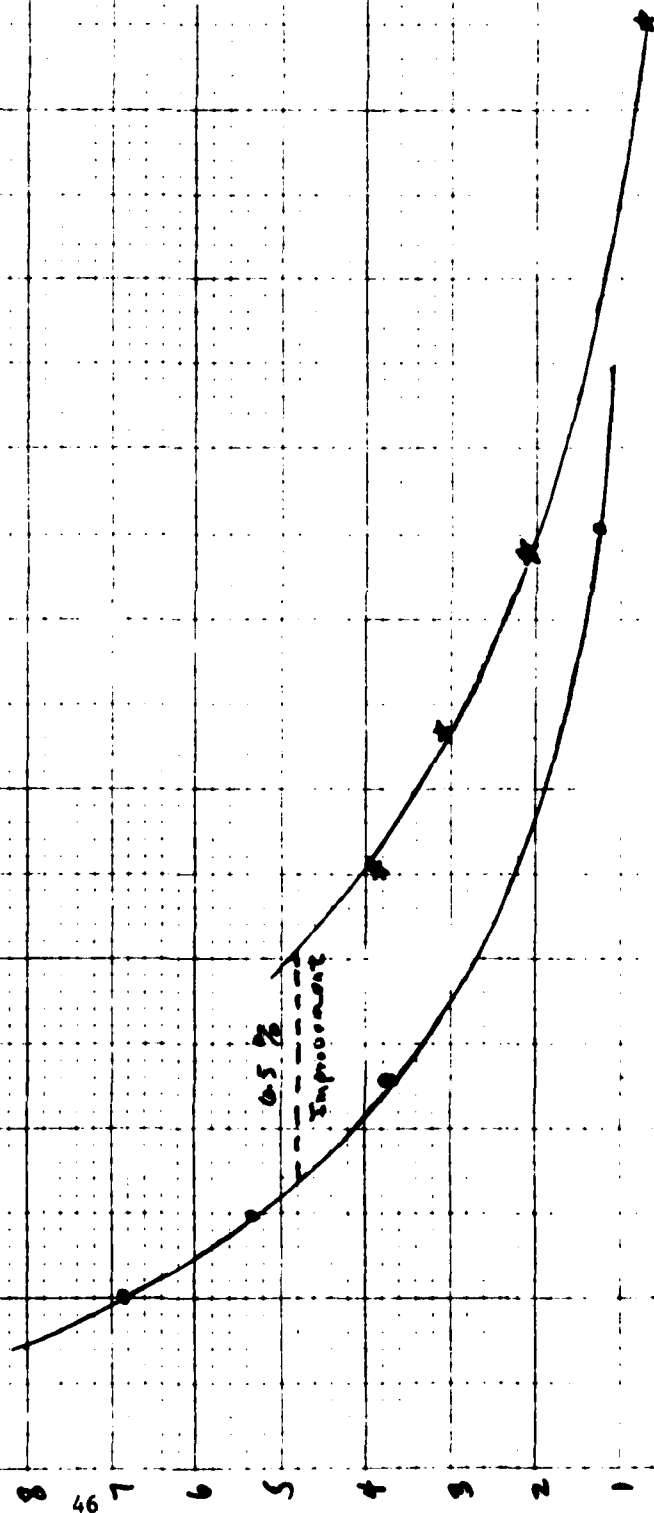
GRAPH # 1

MBVD ⊙   TMTA ×
1794 ⊛   INDN ▵
           DEPM ●

Sample Size = 230

% of Total Reg Not Satisfied

GRAPH #2

Start Class 1   5797 items
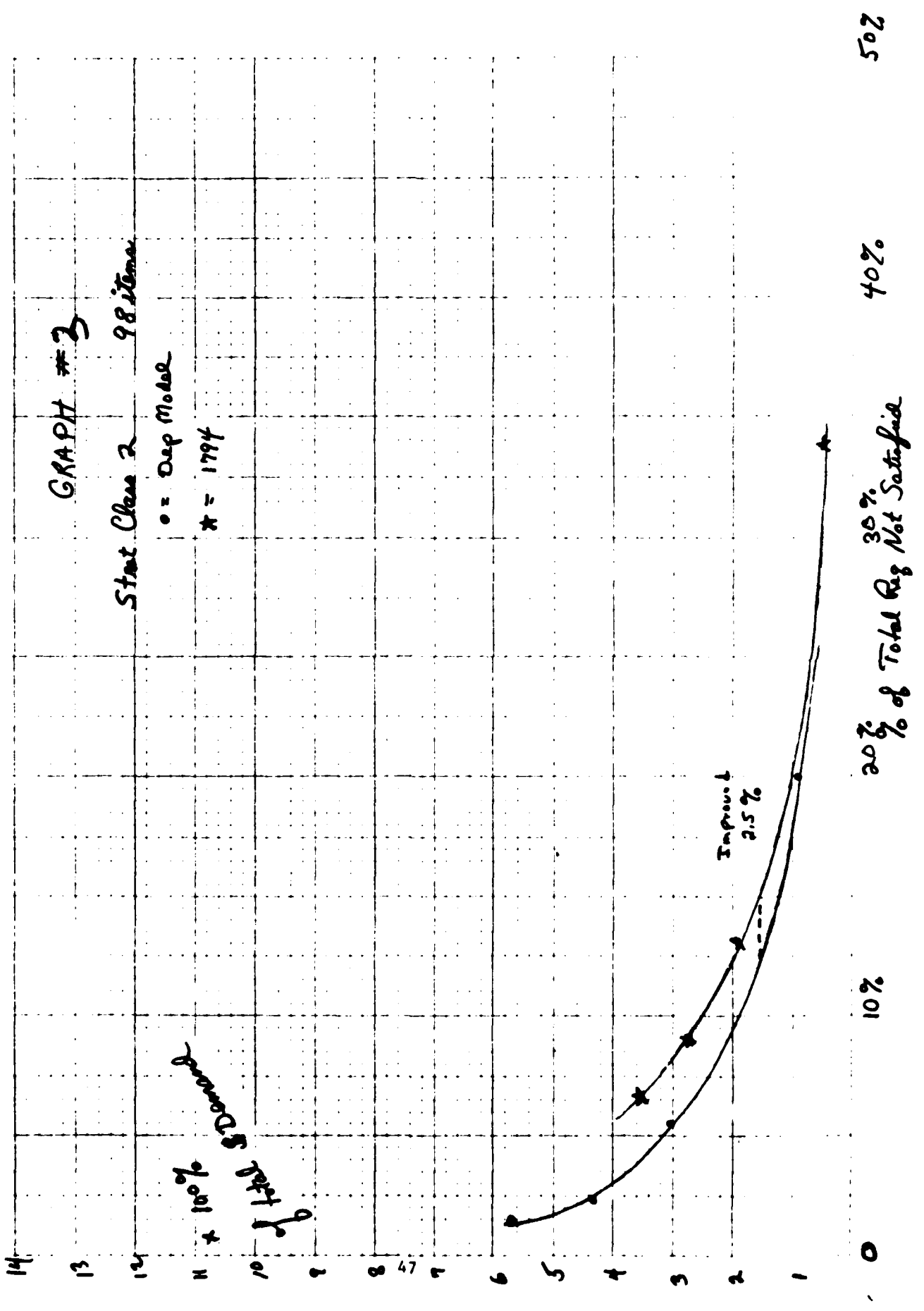
• = Dependent Model
✗ = 1794

% of Total Reg. Net Sales $

65.2
Improvement

% of Total Demand
× 100%

0   10   20   30   40   50

46

GRAPH #3

Stat. Class 2   98 items

• = Dep Model
★ = 1794

% × 100% of Total $ Demand

Improv 2.5%

% of Total Reg Not Satisfied

0   10%   20%   30%   40%   50%

14 13 12 11 10 9 8 47 7 6 5 4 3 2 1

GRAPH #4

Strat Class 4   1943 Items

• = Dependent Model

★ = 1794

% of Total Reg. Net Sets ful

100% $ Dual
× Total of

0   10   20   30   40   50

GRAPH # 5

Strat Class 5   231 items

• = Dependent Model

* = 1794

100 %
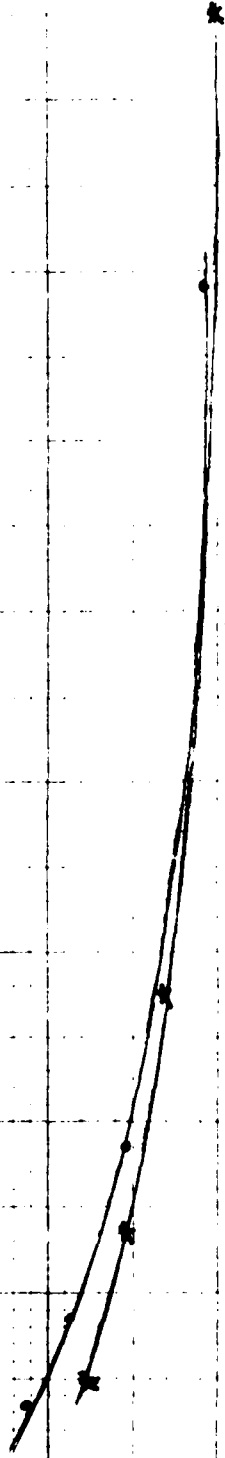of Total $ Demand
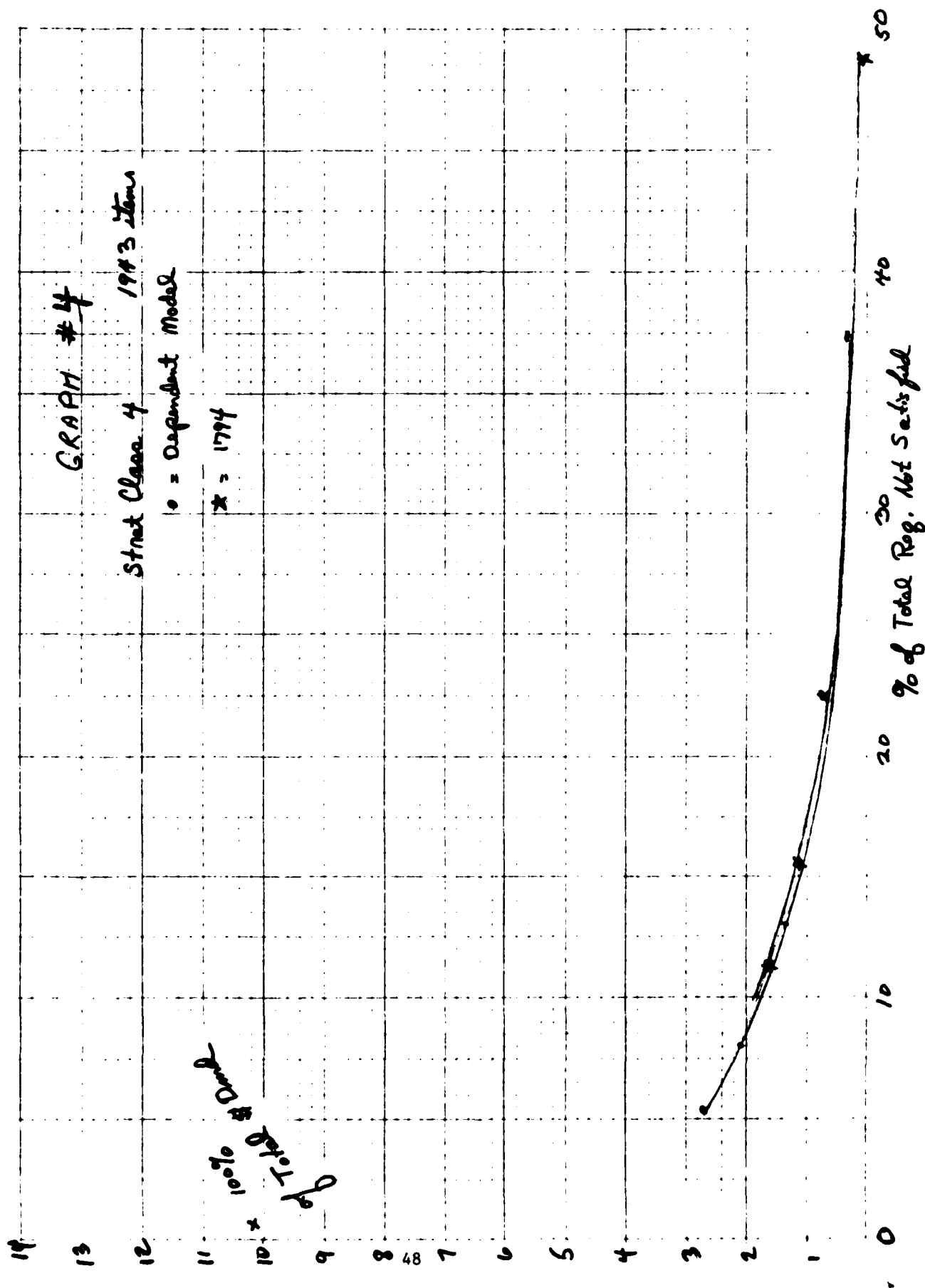
% of Total Req's Not Satisfied

14
13
12
11
10
9
8
49 7
6
5
4
3
2
1

10   20   30   40   50

5.5 Comments on the Dependent Model

Using a heuristic to measure performance, the Dependent Model shows a 6.5% increase in requisitions satisfied over the current forecast method. Additional comments concerning this algorithm are as follows:

Implementation

The data needed for implementation for the Dependent Model are as follows:

(1) The previous two year history is used to determine the requisition class of the series.

(2) A catalog based on empirical estimates is used to determine the expected number of non-zero quarters in the next PLT period. The catalog is a function of requisition class and PLT.

(3) The average demand per non-zero quarter is computed over the item's past history.

(4) The forecast is the product of the average demand per non-zero quarter times the expected number of non-zero quarters for the next PLT.

Refinements

There are several refinements that could be made to the model but weren't explored. Some of these are:

(1) Combine catalog estimates with the item's history.

(2) Use exponential smoothing or a longer base period to determine the requisition class of the series.

(3) Use other estimates to compute the expected size of the next non-zero demand quarter.

Other Considerations

(1) Sensitivity to the catalog. The catalog is computed using estimates of the probability of seeing a non-zero demand quarter. This probability is directly related to the requisition class of the item and may not be a characteristic of the type of population sampled. If this is true, then the catalog from this report may be used for other types of items. This conjecture has yet to be demonstrated.

(2) Additional testing. The results of this study should be confirmed using a detailed inventory management simulator. Since the management of inactive items is a topic for a future IRO project, this simulation was deferred to this new project.

CHAPTER VI

CONCLUSIONS

6.1 Identification of Inactive Items

Descriptive variables such as the IMPC codes, unit price, maintenance factors, and procurement lead time did not show any direct relationship with the requisition activity of the item. These variables should not be used to identify an inactive item.

The demands per requisition increased as the requisition activity increased. The forecast of demand should consider the requisition frequency of the item.

The interarrival times between non-zero demands appeared to be relatively small even for low requisition items. This could be due to a clustering of requisitions and should be considered when making a forecast.

6.2 Characteristics of Demand

There were some indications that the probability of a non-zero demand quarter was dependent on what happened in the previous quarter. A dependent demand model should be considered along with an independent model.

6.3 Forecast Algorithm Evaluation

The decomposition type forecast algorithm performed better than the models currently being used for inactive items. The Dependent Algorithm performed best with an estimated 5.5% increase in performance for the low demand items. This increase in performance indicates that additional work with the model may be fruitful.

6.4 Future Work

a. Develop new management policies incorporating the results from this study. Develop a simulator to test these policies.

b. Make refinements as mentioned in 5.5 to the Dependent Model and test in simulator from above.

c. Determine the applicability of the catalog produced in this study for use with inactive items in general.

REFERENCES

1   Box & Jenkins, <u>Time Series Analysis: Forecasting & Control</u>, Holden-Day, 1970.

2   Cohen, Martin, "Alpha 4140.39 Simulator," AMC Inventory Research Office, May 1973, AD762348.

3   Cohen, Martin, "Demand Forecasting With Program Factors," AMC Inventory Research Office, September 1975, ADA017858.

4   Conover, W. J., <u>Practical Non-Parametric Statistics</u>, John Wiley & Sons, Inc., 1971.

5   Croston, J. D., "Forecasting and Stock Control for Intermittent Demands," Operations Research Quarterly (UK), September 1972.

6   Gotwals, E. P. & A. Hutchison, "Use of End Item Age in Demand Forecasting," DRC Inventory Research Office, December 1976, ADA037585.

7   Orr, Donald, "Demand Forecasts Using Process Models and Item Class Parameters: Application of Ancillary Variables," DRC Inventory Research Office, April 1976, ADA026081.

8   Sweet, A. L., "Forecasting the Demand for Steel," Report No. 102, Purdue Laboratory for Applied Industrial Control, July 1978.

9   Tiao, G. C., and M. R. Grupe, "Hidden Periodic Autoregressive Moving Average Models in Time Series Data," Technical Report No. 572, Dept of Statistics, University of Wisconsin-Madison, July 1979.

10. Gotwals, E. P. and D. Orr, "Integrated Forecasting Techniques for Secondary Items Classes - Part I, Active Items," US Army Inventory Research Office, to be published Sep 80.

11 Gotwals, E. P., "Evaluation of Forecast Methods in an Inventory Management System (An Empirical Study)," US Army Inventory Research Office, to be published Nov 80.

APPENDIX

STATISTICAL ERROR MEASURES COMPARING VARIOUS FORECAST

ALGORITHMS WITH ZERO FORECAST ON A CLASS OF LOW

DEMAND ITEMS (ST1)

53

STATISTICAL RESULTS

(STRAT CLASS 1)

*	ERROR		RELATIVE ERROR		MAD		REL MAD		REL MSE	
	1st Qtr	PLT	1st Qtr	PLT	1st Qtr	PLT	1st Qtr	PLT	1st Qtr	PLT
1794	.366	2.15	1.58	1.79	1.43	3.36	2.73	2.39	29.7	25.52
KAL	1.95	6.67	7.58	7.46	2.84	7.45	8.41	7.75	168.	152.43
TRIG	.84	3.35	3.11	3.18	1.85	4.38	4.12	3.63	34.82	27.45
MED	-.34	-.02	-.18	.07	.89	1.66	1.11	.90	6.57	3.22
MOVD	.48	2.64	1.80	2.06	1.55	3.82	2.92	2.64	35.11	32.95
ZERO	-.69	-1.19	-.74	-.55	.69	1.19	.74	.54	5.80	1.97

*These forecast methods are described on the next page.

54

Forecast Methods

The five basic forecast algorithm tested were of the form of a weighted moving average (1794), moving median (MED), Kalman filter (KAL), a variable base moving average utilizing a tracking signal (TRIG) and a moving average on demand (MOV-D). The underlying structure of these algorithms are as follows:

(1794):

> This is the current Army forecasting procedure which is a weighted eight quarter moving average with weights proportioned to quarterly flying hours.

(MEDIAN):

> This is the median of the last four quarters.

(Kalman Filter):

> For a general class of statistical processes, this is a minimum mean squared error algorithm which is structurally similar to exponential smoothing where the smoothing weights are variable and updated.

(TRIG Tracking Signal):

> This method switches between a weighted four and eight quarter moving average where the weights are parameters determining the amount of additional weight to give the current observations.

(MOV-D):

> This is an eight quarter moving average on demand.

DISTRIBUTION

COPIES

1	Deputy Under Sec'y of the Army, ATTN: Office of Op Resch
1	Asst Sec'y of the Army (I,L&FM), Pentagon, Wash., DC 20310
	Headquarters, US Army Materiel Development & Readiness Command
1	DRCPA-S
1	DRCMS
1	DRCDMR
1	DRCPS
1	DRCPS-P, ATTN: Ms. Lamb
1	DRCPS-S
1	DRCMM
1	DRCMM-RS
1	DRCMM-M
1	DRCMM-E
1	DRCRE
1	Dep Chf of Staff for Logistics, ATTN: DALO-SMS, Pentagon, Wash., DC 20310
1	Dep Chf of Staff for Logistics, ATTN: DALO-SML, Pentagon, Wash., DC 20310
1	Commandant, US Army Logistics Mgt Center, Ft. Lee, VA 23801
1	Office, Asst Sec'y of Defense, ATTN: MRA&L-SR, Pentagon, Wash., DC 20310
2	Defense Logistics Studies Info Exchange, DRXMC-D
10	Defense Technical Information Center, Cameron Station, Alexandria,
	Commander, USA Armament Materiel Readiness Cmd, Rock Island, I'. 61201
1	ATTN: DRSAR-MM
1	ATTN: DRSAR-SA
	Commander, USA Communications & Electronics Materiel Readiness Cmd, Ft. Monmouth, NJ 07703
1	ATTN: DRSEL-MM
1	ATTN: DRSEL-SA
	Commander, USA Missile Command, Redstone Arsenal, AL 35809
1	ATTN: DRSMI-S
1	ATTN: DRSMI-D
	Commander, USA Troop Support & Aviation Materiel Readiness Command, St. Louis, MO
1	ATTN: DRSTS-SP
1	ATTN: DRSTS-SPSS _1_ DRSTS-BA(1)
	Commander, US Army Tank-Automotive Command, Warren, MI 48090
1	ATTN: DRSTA-F
1	ATTN: DRSTA-S
1	Commander, US Army Armament Research & Development Command, ATTN: DRDAR-SE, Dover, NJ 07801
1	Commander, US Army Aviation Research & Development Command, 4300 Goodfellow Blvd., St. Louis, MO 63120
1	Commander, US Army Communications Research & Development Command, ATTN: DRSEL-SA, Ft. Monmouth, NJ 07703
1	Commander, US Army Electronics Research & Development Command, ATTN: DRDEL-AP, Adelphi, MD 20783

COPIES	
1	Commander, US Army Mobility Equipment Research & Development Cmd, ATTN: DRDME-O, Ft. Belvoir, VA 22060
1	Commander, US Army Natick Research & Development Command, ATTN: DRXNM-O, Natick, MA 01760
1	Commander, US Army Logistics Center, Ft. Lee, VA 23801
1	Commander, US Army Logistics Evaluation Agency, New Cumberland Army Depot, New Cumberland, PA 17070
1	Commander, US Army Depot Systems Command, Chambersburg, PA 17201
1	Commander, US Air Force Logistics Cmd, WPAFB, ATTN: AFLC/XRS, Dayton, Ohio 45433
1	US Navy Fleet Materiel Support Office, Naval Support Depot, Mechanicsburg, PA 17055
1	Mr. James Prichard, Navy SEA Systems Cmd, ATTN: PMS 3061, Dept of US Navy, Wash., DC 20362
1	George Washington University, Inst of Management Science & Engr., 707 22nd St., N.W., Wash., DC 20006
1	Naval Postgraduate School, ATTN: Dept of Opns Anal, Monterey, CA 93940
1	Commander, US Army Aviation R&D Cmd, ATTN: DRDAV-BD (Marl Glenn), 4300 Goodfellow Blvd., St. Louis, MO 63120
1	Air Force Institute of Technology, ATTN: SLGQ, Head Quantitative Studies Dept., Dayton, OH 43433
1	US Army Military Academy, West Point, NY 10996
1	Librarian, Logistics Mgt Inst., 4701 Sangamore Rd., Wash.,DC 20016
1	University of Florida, ATTN: Dept of Industrial Systems Engr., Gainesville, FL 32601
1	RAND Corp., ATTN: S. M. Drezner, 1700 Main St., Santa Monica, CA 90406
1	US Army Materiel Systems Analysis Activity, ATTN: DRXSY-CL, Aberdeen Proving Ground, MD 21005
1	Commander, US Army Logistics Center, ATTN: Concepts & Doctrine Directorate, Ft. Lee, VA 23801
1	ALOG Magazine, ATTN: Tom Johnson, USALMC, Ft. Lee, VA 23801
1	Commander, USDRC Automated Logistics Mgt Systems Activity, P.O. Box 1578, St. Louis, MO 63188
1	Director, DARCOM Logistics Systems Support Agency, Letterkenny Army Depot, Chambersburg, PA 17201
1	Commander, Materiel Readiness Support Activity, Lexington, KY 40507
1	Director, Army Management Engineering Training Agency, Rock Island Arsenal, Rock Island, IL 61202
1	Defense Logistics Agcy, ATTN: DLA-LO, Cameron Sta, Alexandria, VA 22314
1	Dep Chf of Staff (I&L), HQ USMC-LMP-2, ATTN: MAJ Sonneborn, Jr., Wash., DC 20380
1	Commander, US Army Depot Systems Command, Letterkenny Army Depot, ATTN: DRSDS-LL, Chambersburg, PA 17201
1	Logistics Control Activity, Presidio of San Francisco, CA 94120
1	HQ, Dept of the Army, (DASG-HCL-P), Wash., DC 20314
1	Operations Research Center, 3115 Etcheverry Hall, University of California, Berkeley, CA 94720

COPIES	
1	Dr. Jack Muckstadt, Dept of Industrial Engineering & Operations Research, Upson Hall, Cornell University, Ithaca, NY 14890
1	Prof Herbert P. Galliher, Dept of Industrial Engineering, University of Michigan, Ann Arbor, MI 48104
1	Scientific Advisor, ATCL-SCA, Army Logistics Center, Ft. Lee, VA 23801
1	Commandant, USA Armor School, ATTN: MAJ Harold E. Burch, Leadership Dept, Ft. Knox, KY 40121
1	Prof Robert M. Stark, Dept of Stat & Computer Sciences, University of Delaware, Newark, DE 19711
1	Prof E. Gerald Hurst, Jr., Dept of Decision Science, The Wharton School, University of Penna., Phila., PA 19174
1	Logistics Studies Office, DRXMC-LSO, ALMC, Ft. Lee, VA 23801
1	Procurement Research Office, DRXMC-PRO, ALMC, Ft. Lee, VA 23801
1	Dept of Industrial Engr. & Engr. Management, Stanford University, Stanford, CA 94305
1	Commander, US Army Communications Command, ATTN: Dr. Forry, CC-LOG-LEO, Ft. Huachuca, AZ 85613
1	Commander, US Army Test & Evaluation Cmd, ATTN: DRSTE-SY, Aberdeen Proving Ground, MD 21005
1	Prof Harvey M. Wagner, Dean, School of Business Adm, University of North Carolina, Chapel Hill, NC 27514
1	DARCOM Intern Training Center, Red River Army Depot, Texarkana, Tx 75501
1	Prof Leroy B. Schwarz, Dept of Management, Purdue University, Krannert Bldg, West Lafayette, Indiana 47907
1	US Army Training & Doctrine Command, Ft. Monroe, VA 23651
1	US General Accounting Office, ATTN: Mr. J. Morris, Rm 5840, 441 G. St., N.W., Wash., DC 20548
1	Operations & Inventory Analysis Office, NAVSUP (Code 04A) Dept of Navy, Wash., DC 20376
1	US Army Research Office, ATTN: Robert Launer, Math. Div., P.O. Box 12211, Research Triangle Park, NC 27709
1	Prof William P. Pierskalla, 3641 Locust Walk CE, Philadelphia, PA 19104
1	US Army Materiel Systems Analysis Activity, ATTN: DRXSY-MP, Aberdeen Proving Ground, MD 21005
	Air Force Logistics Management Center, Gunter Air Force Station, AL 36144
1	ATTN: AFLMC/LGY
1	ATTN: AFLMC/XRP, Bldg. 205
1	Engineer Studies Center, 6500 Brooks Lane, Wash., DC 20315
1	US Army Materiel Systems Analysis Activity, ATTN: Mr. Herbert Cohen, DRXSY-MP, Aberdeen Proving Ground, MD 21105
1	Dr. Carl Weisman, C.A.C.I., 1815 North Fort Myer Drive, Arlington,VA 22209
1	Commander, US Army Missile Cmd, ATTN: Ray Dotson, DRSMI-DS, Redstone Arsenal, AL 35809
1	Prof Peter Kubat, Graduate School of Mgmt, University of Rochester, Rochester, NY 14627